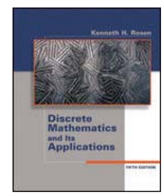
Chapter 7: Relations

- Relations(7.1)
- n-any Relations & their Applications (7.2)



Relations (7.1)

- Introduction
  - Relationship between a program and its variables
  - Integers that are congruent modulo k
  - Pairs of cities linked by airline flights in a network

Relations (7.1) (cont.)

- Relations & their properties

  – Definition 1

  > Let A and B be sets. A binary relation from A to B is a subset of A * B.

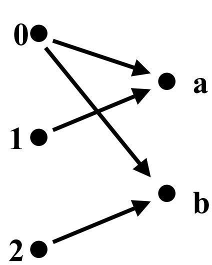
  In other words, a binary relation from A to B is a set R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B.

Relations (7.1) (cont.)

– Notation:

$aRb \Leftrightarrow (a, b) \in R$

$a \not{R} b \Leftrightarrow (a, b) \notin R$



R	a	b
0	X	X
1	X	
2		X

Relations (7.1) (cont.)

– Example:
A = set of all cities
B = set of the 50 states in the USA
Define the relation R by specifying that (a, b) belongs to R if city a is in state b.

$$\left.\begin{array}{l} (Boulder, Colorado) \\ (Bangor, Maine) \\ (Ann\ Arbor, Michigan) \\ (Cupertino, California) \\ Red\ Bank, New\ Jersey) \end{array}\right\} \text{are in } R.$$

Relations (7.1) (cont.)

- Functions as relations
  - The graph of a function f is the set of ordered pairs (a, b) such that b = f(a)
  - The graph of f is a subset of A * B ⇒ it is a relation from A to B
  - Conversely, if R is a relation from A to B such that every element in A is the first element of exactly one ordered pair of R, then a function can be defined with R as its graph

Relations (7.1) (cont.)

- Relations on a set
  - Definition 2

  > A relation on the set A is a relation from A to A.

  - Example: A = set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}

    Solution: Since (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b

    R = {(1,1), (1,2), (1.3), (1.4), (2,2), (2,4), (3,3), (4,4)}

Relations (7.1) (cont.)

- Properties of Relations
  - Definition 3

  > A relation R on a set A is called reflexive if (a, a) $\in$ R for every element a $\in$ A.

Relations (7.1) (cont.)

– Example (a): Consider the following relations on {1, 2, 3, 4}

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Which of these relations are reflexive?

Relations (7.1) (cont.)

Solution:

R_3 and R_5: reflexive $\Leftarrow$ both contain all pairs of the form (a, a): (1,1), (2,2), (3,3) & (4,4).

R_1, R_2, R_4 and R_6: not reflexive $\Leftarrow$ not contain all of these ordered pairs. (3,3) is not in any of these relations.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (7.1) (cont.)

– Definition 4:

A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R, for all a, b ∈ A.
A relation R on a set A such that (a, b) ∈ R and (b, a) ∈ R only if a = b, for all a, b ∈ A, is called antisymmetric.

Relations (7.1) (cont.)

– Example: Which of the relations from example (a) are symmetric and which are antisymmetric?

Solution:

❖ R_2 & R_3: symmetric ⇐ each case (b, a) belongs to the relation whenever (a, b) does.
For R_2: only thing to check that both (1,2) & (2,1) belong to the relation
For R_3: it is necessary to check that both (1,2) & (2,1) belong to the relation.
None of the other relations is symmetric: find a pair (a, b) so that it is in the relation but (b, a) is not.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (7.1) (cont.)

Solution (cont.):

- ❖ R_4, R_5 and R_6: antisymmetric ⇐for each of these relations there is no pair of elements a and b with $a \neq b$ such that both (a, b) and (b, a) belong to the relation.
None of the other relations is antisymmetric.: find a pair (a, b) with $a \neq b$ so that (a, b) and (b, a) are both in the relation.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (7.1) (cont.)

– Definition 5:

A relation R on a set A is called transitive if whenever (a, b) ∈ R and (b,c) ∈ R, then (a, c) ∈ R, for all a, b, c ∈ R.

Relations (7.1) (cont.)

– Example: Which of the relations in example (a) are transitive?

❖ R_4 , R_5 & R_6 : transitive ⇐ verify that if (a, b) and (b, c) belong to this relation then (a, c) belongs also to the relation
R_4 transitive since (3,2) and (2,1), (4,2) and (2,1), (4,3) and (3,1), and (4,3) and (3,2) are the only such sets of pairs, and (3,1) , (4,1) and (4,2) belong to R_4.
Same reasoning for R_5 and R_6.

❖ R_1 : not transitive ⇐ (3,4) and (4,1) belong to R_1, but (3,1) does not.

❖ R_2 : not transitive ⇐ (2,1) and (1,2) belong to R_2, but (2,2) does not.

❖ R_3 : not transitive ⇐ (4,1) and (1,2) belong to R_3, but (4,2) does not.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (7.1) (cont.)

- Combining relations

  – Example:

  Let A = {1, 2, 3} and B = {1, 2, 3, 4, }. The relations R_1 = {(1,1), (2,2), (3,3)} and R_2 = {(1,1), (1,2), (1,3), (1,4)} can be combined to obtain:

  $R_1 \cup R_2$ = {(1,1), (1,2), (1,3), (1,4), (2,2), (3,3)}
  $R_1 \cap R_2$ = {(1,1)}
  $R_1 - R_2$ = {(2,2), (3,3)}
  $R_2 - R_1$ = {(1,2), (1,3), (1,4)}

Relations (7.1) (cont.)

– Definition 6:

Let R be a relation from a set A to a set B and S a relation from B to a set C.
The composite of R and S is the relation consisting of ordered pairs (a, c), where a ∈ A, c ∈ C, and for which there exists an element b ∈ B such that (a, b) ∈ R and (b, c) ∈ S. We denote the composite of R and S by S ∘ R.

Relations (7.1) (cont.)

– Example: What is the composite of the relations R and S where R is the relation from {1,2,3} to {1,2,3,4} with R = {(1,1), (1,4), (2,3), (3,1), (3,4)} and S is the relation from {1,2,3,4} to {0,1,2} with S = {(1,0), (2,0), (3,1), (3,2), (4,1)}?

Solution: S ∘ R is constructed using all ordered pairs in R and ordered pairs in S, where the second element of the ordered in R agrees with the first element of the ordered pair in S.
For example, the ordered pairs (2,3) in R and (3,1) in S produce the ordered pair (2,1) in S ∘ R. Computing all the ordered pairs in the composite, we find

S ∘ R = ((1,0), (1,1), (2,1), (2,2), (3,0), (3,1)}

N-ary Relations & their Applications (7.2)

- Relationship among elements of more than 2 sets often arise: n-ary relations
- Airline, flight number, starting point, destination, departure time, arrival time

N-ary Relations & their Applications (7.2) (cont.)

- N-ary relations
  - Definition 1:

    Let A_1, A_2, …, A_n be sets. An n-ary relation on these sets is a subset of $A_1 * A_2 * \ldots * A_n$ where A_i are the domains of the relation, and n is called its degree.

  - Example: Let R be the relation on N * N * N consisting of triples (a, b, c) where a, b, and c are integers with $a<b<c$. Then $(1,2,3) \in R$, but $(2,4,3) \notin R$. The degree of this relation is 3. Its domains are equal to the set of integers.

N-ary Relations & their Applications (7.2) (cont.)

- Databases & Relations
  - Relational database model has been developed for information processing
  - A database consists of records, which are n-tuples made up of fields
  - The fields contains information such as:
    - Name
    - Student #
    - Major
    - Grade point average of the student

N-ary Relations & their Applications (7.2) (cont.)

- The relational database model represents a database of records or n-ary relation
- The relation is R(Student-Name, Id-number, Major, GPA)

N-ary Relations & their Applications (7.2) (cont.)

– Example of records

(Smith, 3214, Mathematics, 3.9)
(Stevens, 1412, Computer Science, 4.0)
(Rao, 6633, Physics, 3.5)
(Adams, 1320, Biology, 3.0)
(Lee, 1030, Computer Science, 3.7)

N-ary Relations & their Applications (7.2) (cont.)

TABLE A: Students

Students Names	**ID #**	**Major**	**GPA**
Smith	3214	Mathematics	3.9
Stevens	1412	Computer Science	4.0
Rao	6633	Physics	3.5
Adams	1320	Biology	3.0
Lee	1030	Computer Science	3.7

N-ary Relations & their Applications (7.2) (cont.)

- Operations on n-ary relations
  - There are varieties of operations that are applied on n-ary relations in order to create new relations that answer eventual queries of a database
  - Definition 2:

> Let R be an n-ary relation and C a condition that elements in R may satisfy. Then the selection operator s_C maps n-ary relation R to the n-ary relation of all n-tuples from R that satisfy the condition C.

N-ary Relations & their Applications (7.2) (cont.)

- Example:

  if s_C = “Major = “computer science” $\wedge$ GPA > 3.5” then the result of this selection consists of the 2 four-tuples:

  (Stevens, 1412, Computer Science, 4.0)
  (Lee, 1030, Computer Science, 3.7)

N-ary Relations & their Applications (7.2) (cont.)

– Definition 3:

The projection $P_{i_1, i_2, \ldots, i_m}$ maps the n-tuple (a_1, a_2, …, a_n) to the m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$ where m ≤ n.

In other words, the projection $P_{i_1, i_2, \ldots, i_m}$ deletes n – m of the components of n-tuple, leaving the i_1th, i_2th, …, and i_mth components.

N-ary Relations & their Applications (7.2) (cont.)

– Example: What relation results when the projection $P_{1,4}$ is applied to the relation in Table A?

Solution: When the projection $P_{1,4}$ is used, the second and third columns of the table are deleted, and pairs representing student names and GPA are obtained. Table B displays the results of this projection.

TABLE B: GPAs

Students Names	GPA
Smith	3.9
Stevens	4.0
Rao	3.5
Adams	3.0
Lee	3.7

N-ary Relations & their Applications (7.2) (cont.)

– Definition 4:

Let R be a relation of degree m and S a relation of degree n. The join $J_p(R,S)$, where $p \leq m$ and $p \leq n$, is a relation of degree $m + n - p$ that consists of all $(m + n - p)$-tuples $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$, where the m-tuple $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p)$ belongs to R and the n-tuple $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$ belongs to S.

N-ary Relations & their Applications (7.2) (cont.)

– Example: What relation results when the operator J_2 is used to combine the relation displayed in tables C and D?

TABLE C: Teaching Assignments

Professor	Dpt	Course #
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE D: Class Schedule

Dpt	Course #	Room	Time
Computer Science	518	N521	2:00 PM
Mathematics	575	N502	3:00 PM
Mathematics	611	N521	4:00 PM
Physics	544	B505	4:00 PM
Psychology	501	A100	3:00 PM
Psychology	617	A110	11:00 AM
Zoology	335	A100	9:00 AM
Zoology	412	A100	8:00 AM

N-ary Relations & their Applications (7.2) (cont.)

Solution: The join J_2 produces the relation shown in Table E

Table E: Teaching Schedule

Professor	Dpt	Course #	Room	Time
Cruz	Zoology	335	A100	9:00 AM
Cruz	Zoology	412	A100	8:00 AM
Farber	Psychology	501	A100	3:00 PM
Farber	Psychology	617	A110	11:00 AM
Grammer	Physics	544	B505	4:00 PM
Rosen	Computer Science	518	N521	2:00 PM
Rosen	Mathematics	575	N502	3:00 PM

N-ary Relations & their Applications (7.2) (cont.)

– Example: We will illustrate how SQL (Structured Query Language) is used to express queries by showing how SQL can be employed to make a query about airline flights using Table F. The SQL statements

```
SELECT departure_time
FROM Flights
WHERE destination = 'Detroit'
```

are used to find the projection P_5 (on the departure_time attribute) of the selection of 5-tuples in the flights database that satisfy the condition: destination = 'Detroit'. The output would be a list containing the times of flights that have Detroit as their destination, namely, 08:10, 08:47, and 9:44.

N-ary Relations & their Applications (7.2) (cont.)

Table F: Flights

Airline	Flight #	Gate	Destination	Departure time
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08"47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44